Quiz 1: MCMC and Variational Inference

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EXERCISE 1 In this question, we assume that

$$\pi : x \mapsto \pi(x) = \frac{\mathrm{e}^{-x^2/2}}{\sqrt{2\pi}}$$

is the density of the standard Gaussian distribution $N(0,1)$. Which of the following assertions is true?

(a) we have $\pi(\mathrm{d}x)\delta_{2x}(\mathrm{d}y) = \pi(\mathrm{d}y)\delta_{y/2}(\mathrm{d}x)$

(b) if $X \sim N(0,1)$, the random variable $Y = X/2$ has the density $y \mapsto \frac{\pi(y/2)}{2}$

(c) ✓ if $(X,Y) \sim \pi(\mathrm{d}x)\delta_{2x}(\mathrm{d}y)$, then, [$Y \sim N(0,4)$ and $Y = 2X$]

(d) ✓ we have $\pi(\mathrm{d}x)\delta_x(\mathrm{d}y) = \pi(\mathrm{d}y)\delta_y(\mathrm{d}x)$

(e) we have $\pi(\mathrm{d}x)\delta_{2x}(\mathrm{d}y) = \pi(\mathrm{d}y)\delta_{2y}(\mathrm{d}x)$

(f) ✓ if $X \sim N(0,1)$, the random variable $Y = X/2$ has the density $y \mapsto 2\pi(2y)$

(g) if $(X,Y) \sim \pi(\mathrm{d}x)\delta_{2x}(\mathrm{d}y)$, then, [$Y \sim N(0,2)$ and $Y = 2X$]

EXERCISE 2 Suppose that we have two positive densities π and ϕ on $\mathbb{R}$ with respect to the lebesgue measure such that

$$\pi(x) \geq \varepsilon\phi(x)\,, \quad \forall x \in \mathbb{R}.$$

We also assume that we are able to draw according to ϕ.

Define the Markov kernels

$$P_0(x,\mathrm{d}y) = \frac{\varepsilon\phi(x)}{\pi(x)}\phi(y)\mathrm{d}y + \left[1 - \frac{\varepsilon\phi(x)}{\pi(x)}\right]\delta_x(\mathrm{d}y)$$

$$P_1(x,\mathrm{d}y) = \alpha(x,y)\phi(y)\mathrm{d}y + \left[1 - \int \alpha(x,z)\phi(z)\mathrm{d}z\right]\delta_x(\mathrm{d}y) \quad \text{with} \quad \alpha(x,y) = \min\left(\frac{\pi(y)\phi(x)}{\pi(x)\phi(y)}, 1\right).$$

Which of the following assertions is true?

(a) ✓ P_0 is π-reversible.

(b) P_0 is ϕ-reversible.

(c) ✓ P_1 is π-reversible.

(d) ✓ P_0 has a unique invariant probability measure π.

(e) ϕ is an invariant distribution for P_0.

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