Quiz 4: MCMC and Variational Inference

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Nom:

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Algorithm 1 Input: $x_0 \in \mathbb{R}$, $\sigma > 0$, $a, a' \in \mathbb{R}$, $\alpha \in (0,1)$.

$X_0 = x_0$.
for $k \geqslant 0$ **do**
  Independently sample $\eta_{k+1} \sim \mathcal{N}(0, \sigma^2)$ and U_{k+1} from a uniform distribution on $[0,1]$.
  if $U_{k+1} \leqslant \alpha$ **then**
    $X_{k+1} = aX_k + \eta_{k+1}$
  else
    $X_{k+1} = a'X_k + \eta_{k+1}$
  end if
end for

EXERCISE 1 Let (X_k) be a Markov Chain generated by Algorithm 1. We will denote P its Markov kernel and $V(x) = x^2 + 1$. Which of the following statements is true?

a ✓ P is irreducible with respect to the Lebesgue measure.

b ✓ For $\alpha = 3/4$, the drift condition on V is satisfied for $a = 1/2$ and $a' = 1/2$.

c For $\alpha = 1/2$, the drift condition on V is satisfied for $a = 3/2$ and $a' = 1/4$.

d ✓ For all $a, a' \in \mathbb{R}$ the minorizing condition is satisfied.

e For all $a, a' \in \mathbb{R}$, P admits an invariant probability measure π such that $\mathbb{E}_{X\sim\pi}[X^2] < +\infty$.