Quiz 3: MCMC and Variational Inference

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EXERCISE 1 Let $\tilde{\pi}$ be a distribution on a mesurable space $(\mathsf{X}, \mathcal{X})$. Let $P(x, \mathrm{d}y) = \alpha(x)\tilde{\pi}(\mathrm{d}y) + \bar{\alpha}(x)\delta_x(\mathrm{d}y)$ where $\alpha : \mathsf{X} \to (0,1)$ and $\bar{\alpha} = 1 - \alpha$. We assume that there exists a probability measure π such that $\pi P = \pi$. Which of the following statements is true?

a ✓ If h is a bounded measurable function on X such that $Ph(x) = h(x)$, then h is constant.

b ✓ P is irreducible with respect to $\tilde{\pi}$.

c π is not the unique invariant distribution for P.

d ✓ for any measurable function $h : \mathsf{X} \to \mathbb{R}$ such that $\pi(|h|) < \infty$, we have, $\mathbb{P}_\pi - a.s.$, $\lim_{n\to\infty} \frac{1}{n}\sum_{k=0}^{n-1} h(X_k) = \pi(h)$.

e ✓ Assume that $\mathrm{d}\tilde{\pi} = \tilde{\pi}\mathrm{d}\nu$ and $\mathrm{d}\pi = \pi\mathrm{d}\nu$ where the densities π and $\tilde{\pi}$ are strictly positive. Then, there exists a constant M such that $\alpha(x) = M\frac{\tilde{\pi}(x)}{\pi(x)}$.