

Quiz 1: MCMC and Variational Inference

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EXERCISE 1 In this question, we assume that

$$\pi : x \mapsto \pi(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

is the density of the standard Gaussian distribution $N(0, 1)$. Which of the following assertions is true?

- (a) we have $\pi(dx)\delta_{2x}(dy) = \pi(dy)\delta_{y/2}(dx)$
- (b) if $X \sim N(0, 1)$, the random variable $Y = X/2$ has the density $y \mapsto \frac{\pi(y/2)}{2}$
- (c) if $(X, Y) \sim \pi(dx)\delta_{2x}(dy)$, then, $[Y \sim N(0, 4) \text{ and } Y = 2X]$
- (d) we have $\pi(dx)\delta_x(dy) = \pi(dy)\delta_y(dx)$
- (e) we have $\pi(dx)\delta_{2x}(dy) = \pi(dy)\delta_{2y}(dx)$
- (f) if $X \sim N(0, 1)$, the random variable $Y = X/2$ has the density $y \mapsto 2\pi(2y)$
- (g) if $(X, Y) \sim \pi(dx)\delta_{2x}(dy)$, then, $[Y \sim N(0, 2) \text{ and } Y = 2X]$

EXERCISE 2 Suppose that we have two positive densities $\pi \neq \phi$ on $\mathbb{R}$ with respect to the lebesgue measure such that there exists $\varepsilon > 0$ such that

$$\pi(x) \geq \varepsilon \phi(x), \quad \forall x \in \mathbb{R}.$$

We also assume that we are able to draw according to ϕ .

Define the Markov kernels

$$P_0(x, dy) = \frac{\varepsilon \phi(x)}{\pi(x)} \phi(y) dy + \left[1 - \frac{\varepsilon \phi(x)}{\pi(x)} \right] \delta_x(dy)$$

$$P_1(x, dy) = \alpha(x, y) \phi(y) dy + \left[1 - \int \alpha(x, z) \phi(z) dz \right] \delta_x(dy) \quad \text{with} \quad \alpha(x, y) = \min \left(\frac{\pi(y) \phi(x)}{\pi(x) \phi(y)}, 1 \right).$$

Which of the following assertions is true?

- (a) P_0 is π -reversible.
- (b) P_0 is ϕ -reversible.
- (c) P_1 is π -reversible.
- (d) P_0 has a unique invariant probability measure π .
- (e) ϕ is an invariant distribution for P_0 .

¹code: 972x983479afg9xx18sdm837047x347