Simple Linear Regression: exercises

Exercice 1 (Estimation) *By minimising*

$$S(\beta_1, \beta_2) = \sum_{i=1}^{n} (y_i - \beta_1 - \beta_2 x_i)^2.$$

Find the expression of $\hat{\beta}_1$ *and* $\hat{\beta}_2$*.*

Exercice 2 (Bias) *Considering the model*

$$Y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

and under the assumption $\mathcal{H}_2 : \mathbb{E}(\varepsilon_i) = 0$*, for* $i = 1, \cdots, n$ *and* $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = \delta_{ij}\sigma^2$*, evaluate the bias of* $\hat{\beta}_2$ *and* $\hat{\beta}_1$*.*

Exercice 3 (Variance) *Considering the model*

$$Y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

and under the assumption $\mathcal{H}_2 : \mathbb{E}(\varepsilon_i) = 0$*, for* $i = 1, \cdots, n$ *and* $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = \delta_{ij}\sigma^2$*, evaluate the variance of* $\hat{\beta}_2$ *and* $\hat{\beta}_1$*.*

Exercice 4 (Covariance) *Considering the model*

$$Y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

and under the assumption $\mathcal{H}_2 : \mathbb{E}(\varepsilon_i) = 0$*, for* $i = 1, \cdots, n$ *and* $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = \delta_{ij}\sigma^2$*, evaluate the covariance of* $\hat{\beta}_2$ *and* $\hat{\beta}_1$*.*

Exercice 5 (Sum of residuals) *Show that in a simple linear regression model the sum of the residuals is zero.*